J Glob Optim (2009) 44:251–271
DOI 10.1007/s10898-008-9322-x

Optimal transportation policies for production/inventory systems with an unreliable and a reliable carrier

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Received: 23 October 2006 / Accepted: 14 June 2008 / Published online: 4 July 2008

Abstract In this paper, we consider a periodic-review make-to-order production/inventory system with two outbound transportation carriers: One carrier is reliable, the other carrier is less reliable but more economical. The objective is to find the optimal shipping policy that minimizes the total discounted transportation, inventory, and customer waiting costs. Under several scenarios, we characterize the optimal policy and present the structural properties for the optimal control parameters and the key performance measures. Our results provide managerial insights on how a manufacturer can effectively manage its transportation carriers and product shipment. We also discuss several possible extensions of the model.

Keywords Make-to-order · Production/inventory system · Setup cost · Transportation carrier · Optimal policies

1 Introduction

Third party logistics (3PL) is expanding rapidly as more global companies realize the potential cost savings of outsourcing their logistics services. For example, in year 2003, 78% of North American companies used 3PL services and 94% of Western European companies used 3PL services. In North America, 43% of a company's logistics expenses are spent on 3PL

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services (Clyde 2003). By outsourcing the logistics, a manufacturer can focus more on its core competencies to improve profitability.

One of the main problems faced by the manufacturer's logistics is the selection and utilization of the 3PL providers. After outsourcing the transportation to the carriers, one key issue for the manufacturers is how to efficiently manage the carriers. Usually, using the reliable carrier may give rise to a higher shipping cost. On the other hand, utilizing less reliable carrier can reduce the shipping cost but may also incur unexpected disruptions, resulting in delays in shipment. In practice, the carrier usually only has a limited capacity which the manufacturer has to take into consideration when making decisions. The manufacturer's problem is to determine the outsourcing policy, including when and how much to outsource its shipment to each of its transportation carriers so that the total cost over a planning horizon is minimized.

In this paper, we consider a periodic-review make-to-order production/inventory system with one manufacturer and two transportation carriers. One carrier is unreliable in the sense that it can be up or down in a period while the other is reliable and is always available. Customer demands in different periods are i.i.d. random variables. The system incurs inventory holding/customer waiting costs as well as transportation costs. We characterize the optimal shipping policies for the model under several different scenarios. We also present structural results of the performance measures and the optimal control parameters which provide insights to the optimal operations of the system.

This research is motivated by the experience of the first author when he was a summer intern with a company that supplies monitors to Dell and HP Computers. The company has several trucking companies as its potential transportation carriers. When making shipping decisions, the company usually first utilizes the economical carrier to ship its products, and only when that carrier is not available, then it will use the other carriers depending on the emergency of the delivery. In such cases the company has to decide how much to ship, based on the status of stock level and its estimate on the future availability of low cost carriers.

There are several approaches to model a transportation carrier's reliability. One is random yield, which means that some of goods may be damaged during the delivery process; and the other is random delivery leadtimes. Different from the availability of the carrier that we consider in this paper, random yield models examine the impact of uncertainty in the shipping quantity . The interested reader is referred to Yano and Lee (1995) for a detailed review. The stochastic delivery leadtime is related to our model and, in the case the customer waiting cost is linear in leadtime, it can be incorporated into our model by modifying the unit delivery costs for each carrier. Once the shipping quantity for the carrier is determined, the expected waiting costs of customers for the quantity are known. Thus we can consider the unit delivery cost for each carrier as the actual unit delivery cost plus the unit waiting cost for the customer multiplied by the expected delivery leadtime.

In the production/inventory literature, much work has been done on inventory models with transportation consideration. Lee (1986, 1989), extend the classical EOQ model and dynamic lot size model by including step-wise transportation cost structure, respectively. Cetinkaya and Lee (2000) consider a model coordinating inventory and transportation decisions in VMI systems. Wang and Lee (2005) consider two transportation modes with different transportation time and cost. The cheaper mode has longer transportation time. Their objective is to minimize total weighted tardiness plus transportation cost. Toptal and Cetinkaya (2005) study the system-wide cost improvement rates with respect to coordination of inventory replenishment, inbound and outbound transportation costs, and capacity. They present detailed numerical results quantifying the value of coordination with transportation considerations. Bhargave et al. (2006) discuss an E-retailer who uses delivery fees as the lever to influence the consumers' choice of delivery modes. Consumers can choose the fast delivery mode at higher costs or choose the regular mode at lower costs but with longer waiting time. In the literature above, it is all assumed that the transportation carriers are reliable.

There is also a rich literature studying the problem with unreliable suppliers in the supply chain. For deterministic demand models, Parlar and Berkin (1991) study the classic EOQ model with supply disruptions. Moinzadeh and Aggarwal (1997) consider an unreliable production/inventory system with constant production and demand rate with disruptions. The model of Moinzadeh and Aggarwal assumes that the repairing time is constant while the time between disruptions is exponentially distributed random variable. An (s, S) production policy is proposed and the properties of the optimal control parameters are presented.

For stochastic demand models, Arreola-Risa and Decroix (1998) consider stochastic demand inventory system with randomly supply disruption and partially backorder. They obtain the optimal policy parameters for the modified (s, S) policy. The paper by Li (2004) considers a periodic-review inventory model, in which the supplier can be either available or unavailable. The availability of supply is modeled as an alternating renewal process with general distribution for the duration of up and down cycles. They show that the state-dependent base-stock policy is optimal and the optimal base-stock level is nondecreasing if the up cycle has a nondecreasing failure rate. More recently, Tomlin (2006) studies an infinite-horizon periodic-review supply chain with one retailer and two suppliers. One of the suppliers is unreliable and the other one is reliable. Under several different model settings, with linear ordering cost, Tomlin discusses the optimal policies for the following decisions that are crucial to deal with the supply disruptions: Is it better to use single sourcing or dual sourcing strategy and if dual, what is the optimal allocation among suppliers? When and how much lost supply should be rerouted to the alternative supplier? What is the optimal inventory control policy?

The remainder of the paper is organized as follows. In the next section, we describe the general model and introduce the notation. In Sects. 3 and 4 we analyze the model with infinite and finite shipping capacities respectively. We conclude the paper with some discussions of possible extensions in Sect. 5. Some technical proofs are provided in the Appendix and other omitted proofs can be obtained from the authors upon request. Throughout the paper, we use "increasing" and "decreasing" in a non-strict sense, i.e., they represent "nondecreasing" and "nonincreasing", respectively. And for any function $f(x, y)$, we use $f'(x, y)$ to denote the derivative of f with respect to the first variable x.

2 Notation and problem description

Consider a periodic-review make-to-order production/inventory system with one manufacturer and two transportation carriers. The manufacturer's production quantity in each period is based on customer orders and we assume the production leadtime is 1. The demand D_t for each period t is i.i.d. random variable with distribution function $F(\cdot)$. The manufacturer outsources its outbound logistics to two transportation carriers. One of the carriers is unreliable in the sense that it can be up or down in a period and the other one is reliable thus it is always available. The unreliable (resp., reliable) carrier is denoted by U (resp., R). Since we consider a make-to-order production system, the production costs are sunk and we ignore them here for simplicity. There are fixed setup costs whenever the manufacturer schedules a shipment with either one of the carriers. We denote the setup cost for carriers U and R by K_u and K_r, respectively. The unit shipping cost is c_u for carrier U and c_r for carrier R. It is plausible to assume that $K_r \geq K_u$ and $c_r > c_u$ since the reliable carrier usually charges higher price than the unreliable carrier does. The inventory holding cost and customer waiting cost function $h(x)$ is nondecreasing convex with respect to the inventory level x at the end of each period. Because the inventory level at the manufacturer never drops to below 0, we assume $h(x) = 0$ for $x \leq 0$. Here x represents both inventory and the number of outstanding orders because the system is make-to-order. Moreover, both carriers may only have limited shipping capacities. We use C_u to denote the capacity of carrier U when it is available and C_r the capacity for carrier R. Let α denote the discount rate per period, i.e. $0 \leq \alpha < 1$. The objective is to determine the optimal shipping policy that minimizes the total expected discounted cost over a finite planning horizon.

We model the unreliable carrier U as a discrete-time Markov process. This modeling technique is also adopted in Tomlin (2006). Let 0 represent that carrier U is in the up state and $i = 1, 2, \ldots, M$ represent that carrier U has been down for $i - 1$ periods. Let $\lambda(0)$ be the probability that U will remain up in the next period given it is up this period and $\lambda(i)$ be the probability that U will become available after i periods given it has been down for $i - 1$ periods. Given U is in state i at the beginning of a period, it will transit to either state 0 with probability $\lambda(i)$ or state $i + 1$ with probability $1 - \lambda(i)$ at the end of the period. We assume $\lambda(M) = 1$, which means after being down for M periods, U will certainly become available. The rational we model the unreliable carrier as a Markov chain is that the recovering process of a carrier usually only depends on its current status, with some positive probability it can fully recover while it can also evolve to a better state as the recovering process evolves.

Some other notations:

T = the given length of the planning horizon,
x_t = initial inventory level at period t,
i_t = state of unreliable carrier U at the beginning of period t,
y_t = the inventory level after shipping but before demand realizes at period t,
D = the generic one-period demand,
$V_t(x_t, i_t)$ = the minimum total expected discounted cost from period t to T.

The time sequence of events is as follows. First, at the beginning of period t, the products produced at period $t - 1$ are available for shipping. Second, the manufacturer chooses the transportation carrier and determines the shipping quantity $x_t - y_t$. Third, the customer order for current period, D_t, arrives and the manufacturer starts producing the orders. Fourth, at the end of period t, all costs are incurred and state transition happens.

The optimality equation for the general model is

$$\begin{aligned} V_t(x_t, i_t) = \min_{C(i_t)+C_r \geq x_t - y_t \geq 0} \Big\{ & K_u \mathbf{1}(C(i_t) \geq x_t - y_t > 0) + K_r \mathbf{1}(x_t - y_t > C(i_t)) \\ & + c_u(x_t - y_t)^+ + (c_r - c_u)(x_t - y_t - C(i_t))^+ + \mathsf{E}[h(y_t + D)] \\ & + \alpha\big(\lambda(i_t)\mathsf{E}[V_{t+1}(y_t + D, 0)] + (1 - \lambda(i_t))\mathsf{E}[V_{t+1}(y_t + D, i_t + 1)]\big)\Big\}, \end{aligned} \tag{1}$$

where if $i_t = 0$, then $C(i_t) = C_u$, otherwise, $C(i_t) = 0$; $\mathbf{1}(A) = 1$ if A is true, otherwise $\mathbf{1}(A) = 0$, and we assume if $i_t = M$, then $i_{t+1} = 0$ with probability 1. The first two terms inside the optimization operator imply that the firm will not use the high cost reliable carrier unless the capacity of the low-cost unreliable carrier is used up or it is not available. The fourth term represents the extra cost by using the reliable carrier.

In the following two sections, we study different scenarios of the general model described in this section and present the main results of the paper. For simplicity, in all the proofs, we suppress the subscript "t" of x, y and i.

3 Uncapacitated carriers

In this section, we assume that both carriers have infinite transportation capacities and consider two scenarios: no setup costs and setup costs for the carriers.

3.1 No setup cost

We first consider the case that there are no setup costs for both carriers. This type of models is more relevant to the motor carriers industry which consists of high level of variable costs and relatively low fixed costs (Coyle et al. 2000).

Because both carriers have infinite capacities, there is no incentive for the manufacturer to use both carriers when unreliable carrier U is available because it is cheaper. Thus the optimality equation (1) becomes

$$\begin{aligned} V_t(x_t, i_t) = \min_{x_t \geq y_t \geq 0} \{ & c(i_t)(x_t - y_t) + \mathsf{E}[h(y_t + D)] + \alpha\lambda(i_t)\mathsf{E}[V_{t+1}(y_t + D, 0)] \\ & + \alpha(1 - \lambda(i_t))\mathsf{E}[V_{t+1}(y_t + D, i_t + 1)]\}, \end{aligned} \tag{2}$$

where $c(i_t) = c_u$ if $i_t = 0$ and $c(i_t) = c_r$, otherwise. We assume $V_{T+1}(x_{T+1}, i_{T+1}) = c(i_{T+1})x_{T+1}$, that is, at the end of the planning horizon the manufacturer has to fill all the customer orders if there are any.

Define

$$\begin{aligned} J_t(y, i) = & -c(i)y + \mathsf{E}[h(y + D)] + \alpha\lambda(i)\mathsf{E}[V_{t+1}(y + D, 0)] \\ & + \alpha(1 - \lambda(i))\mathsf{E}[V_{t+1}(y + D, i + 1)], \end{aligned} \tag{3}$$

and so

$$V_t(x_t, i_t) = \min_{x_t \geq y_t \geq 0} J_t(y_t, i_t) + c(i_t)x_t.$$

The following lemma can be easily proved by induction on t, so we omit the proof.

Lemma 1 *For any t with $1 \leq t \leq T + 1$ and give i, then $V_t(x, i)$ is convex in x.*

Based on this lemma, for $i = 0, 1, \ldots, M$, we define,

$$s_t(i) = \arg\min_{y \geq 0} J_t(y, i). \tag{4}$$

Note that there may be multiple minimizer of $J_t(y, i)$, in such case, let $s_t(i)$ be the minimum one.

The following result states that the higher the initial inventory level, the higher the optimal inventory and transportation costs over the planning horizon. We omit its proof since it is a special case of Lemma 4 whose proof is given in the Appendix.

Lemma 2 *When there are no setup costs and no capacity constraints for both carriers, $V_t(x, i)$ is increasing in x.*

The following theorem characterizes the optimal shipping policy for the manufacturer.

Theorem 1 *The optimal shipping policy is a state-dependent threshold-type policy. That is, for $t = 1, \ldots, T$ and $i = 0, \ldots, M$,*

$$y_t^*(i) = \begin{cases} s_t(i) & \text{if } \; x_t > s_t(i), \\ x_t & \text{otherwise,} \end{cases}$$

in which $s_t(i)$ is defined as (4).

Proof Note that $J_t(y,i)$ is convex. If $x > s_t(i)$, the manufacturer can reduce its inventory level down to $s_t(i)$ which is the minimizer of the objective cost function. If $x \le s_t(i)$, then the objective function $J_t(y,i)$ is decreasing in y so the optimal policy is not to ship anything. □

Remark 1 If we can consider x_t as the starting backlog level at period t and the ordering quantity must be less than or equal to x_t, then our setting is, mathematically, similar to that of Tomlin (2006), although his model does not include constraint on ordering quantity ($y_t \le x_t$). We note that, however, the remaining results in this section are not reported in Tomlin (2006).

In what follows, we provide some qualitative relationships among the value function, optimal policy and system parameters.

Lemma 3 *For any $i = 0, 1, \dots, M$, $s_t(i)$ is decreasing in t for $t = 1, \dots, T+1$.*

Proof Because of the convexity of $J_t(y,i)$, it is sufficient to show that $J'_{t+1}(y,i) \ge J'_t(y,i)$ for all y and i. We prove this by induction on t. For period $t = T$, since $V_{T+1}(x,i) = c(i)x$,

$$J'_T(y,i) = -c(i) + \mathsf{E}[h(y+D)]' + \alpha\lambda(i)c_u + \alpha(1-\lambda(i))c_r$$

and

$$\begin{aligned} J'_{T-1}(y,i) = &-c(i) + \mathsf{E}[h(y+D)]' + \alpha\lambda(i)\mathsf{E}[V_T(y+D,0)]' \\ &+ \alpha(1-\lambda(i))\mathsf{E}[V_T(y+D,i+1)]'. \end{aligned}$$

As it is not hard to see that $\mathsf{E}[V_T(y+D,0)]' \le c_u$ and $\mathsf{E}[V_T(y+D,i+1)]' \le c_r$, so $J'_T(y,i) \ge J'_{T-1}(y,i)$. For period $t = k+1$, suppose $J'_{k+2}(y,i) \ge J'_{k+1}(y,i)$. Then for period $t = k$, from (3), we know that it is sufficient to show $V'_{k+2}(x,i) \ge V'_{k+1}(x,i)$.

If $x \ge s_{k+1}(i)$, then $x \ge s_{k+2}(i)$ because of the inductive assumption. Hence, $V'_{k+1}(x,i) = V'_{k+2}(x,i) = c(i)$. If $x < s_{k+1}(i)$, then $V'_{k+1}(x,i) = J'_{k+1}(x,i) + c(i)$ and there are two cases. First, $x \ge s_{k+2}(i)$, then $V'_{k+2}(x,i) = c(i) \ge J'_{k+1}(x,i) + c(i)$ because $J'_{k+1}(x,i) \le 0$. Second, $x < s_{k+2}(i)$, then $V'_{k+2}(x,i) = J'_{k+2}(x,i) + c(i) \ge J'_{k+1}(x,i) + c(i)$ by inductive assumption. Thus, $V'_{k+2}(x,i) \ge V'_{k+1}(x,i)$ and so $J'_{k+1}(y,i) \ge J'_k(y,i)$. Therefore, $s_t(i) \ge s_{t+1}(i)$. □

Following this lemma, in the next proposition we further show that the optimal threshold $s_t(i)$ is increasing in i if $\lambda(i)$ is increasing in i for $i > 0$,, i.e., the longer the carrier has been down, the more likely it is to end in the current period. Thus in this case, for the manufacturer, the proposition suggests that it is better to hold more inventory if carrier U has been down for more periods, because it is more likely to become available in the next period.

Proposition 1 *For any $t = 1, \dots, T+1$, if $\lambda(i)$ is increasing in i for $i > 0$, then $s_t(i)$ is increasing in i for $0 \le i \le M$.*

Note that If $\lambda(i+1) \ge \lambda(i)$ for $i = 1, \dots, M-1$, from Proposition 1 and Lemma 3, $s_t(M)$ is the solution of

$$-c_r + \mathsf{E}[h(y+D)]' + \alpha c_u = 0, \tag{5}$$

which can be easily solved.

Moreover, for $i = M-1$, $s_t(M-1)$ is the solution of

$$\begin{aligned}
&J_t'(y, M-1)\\
&\quad = -c_r + \mathsf{E}[h(y+D)]' + \alpha\lambda(M-1)c_u + \alpha(1-\lambda(M-1))\mathsf{E}[V_{t+1}'(y+D, M)]\\
&\quad = -c_r + \mathsf{E}[h(y+D)]' + \alpha\lambda(M-1)c_u + \alpha(1-\lambda(M-1))\Bigg[c_r(1-F(s_t(M)-y))\\
&\qquad + \int_0^{s_t(M)-y} \mathsf{E}[h(y+D+\xi)]' + \lambda(M)\mathsf{E}[V_{t+2}(y+D+\xi, 0)]'dF(\xi)\Bigg]\\
&\quad = -c_r + \mathsf{E}[h(y+D)]' + \alpha\lambda(M-1)c_u + \alpha(1-\lambda(M-1))\Bigg[c_r(1-F(s_t(M)-y))\\
&\qquad + \int_0^{s_t(M)-y} \mathsf{E}[h(y+D+\xi)]' + \alpha c_u dF(\xi)\Bigg] = 0,
\end{aligned}$$

where the last equality follows from $\lambda(M) = 1$ and $s_t(M-1) \geq s_{t+1}(M-1) \geq s_{t+1}(0)$. Similarly, for $i = 1, 2, \ldots, M-2$, $s_t(i)$ is the solution of

$$\begin{aligned}
&-c_r + \mathsf{E}[h(y+D)]' + \alpha\lambda(i)c_u + \alpha(1-\lambda(i))\Bigg[c_r(1-F(s_t(i+1)-y))\\
&\quad + \int_0^{s_t(i+1)-y} \mathsf{E}[h(y+D+\xi)]' + \alpha\lambda(i+1)c_u + \alpha(1-\lambda(i+1))\\
&\quad \mathsf{E}[V_{t+2}(y+D+\xi, i+2)]'dF(\xi)\Bigg] = 0.
\end{aligned}$$

For $i = 0$, $s_t(0)$ is the solution of

$$\begin{aligned}
&(\alpha\lambda(0)-1)c_u + \mathsf{E}[h(y+D)]' + \alpha(1-\lambda(0))\Bigg[c_r(1-F(s_t(1)-y))\\
&\quad + \int_0^{s_t(1)-y} \mathsf{E}[h(y+D+\xi)]' + \alpha\lambda(1)c_u + \alpha(1-\lambda(1))\mathsf{E}[V_{t+2}(y+D+\xi, 2)]'dF(\xi)\Bigg] = 0.
\end{aligned}$$

Based on the proceeding discussion, the optimal control parameter $s_t(i), i = 0, 1, \ldots, M$, can be computed recursively.

Remark 2 $\mathsf{E}[h'(D)]$ is the minimum marginal cost to have one more customer to wait, αc_u is the discounted unit delivery cost in the next period by using the unreliable carrier if it is available, and c_r is the unit cost to deliver now by using the reliable carrier. If $\mathsf{E}[h'(D)] + \alpha c_u \geq c_r$, then $s_t(i) = 0$ for all i and it is optimal for the manufacturer to ship every unit of the product to the customers right after it is produced and hold zero inventory since it is more cost effective. If $c_r > \mathsf{E}[h'(D)] + \alpha c_u \geq c_u$, then the optimal policy is that the manufacturer ships all of its on-hand inventory if carrier U is available, otherwise, follows the threshold type policy for carrier R.

In the following proposition, we investigate how the value functions and optimal solutions depend on M which is the maximum number of periods that the unreliable carrier needs to recover from down state. Clearly, $V_t(x, i)$ and $J_t(y, i)$ depend on M through $\lambda(i)$ which is written as $\lambda(i, M)$ in the proposition to emphasize the dependence on M. Let $M_2 \geq M_1$ and specifically, we are interested to see how $V_t(x, i)$ and $s_t(i)$ change when $M = M_1$ increases to $M = M_2$.

Proposition 2 *For $i = 0, 1, \ldots, M_1$, if $M_1 \leq M_2$ and $\lambda(i, M_1) \geq \lambda(i, M_2)$ and $\lambda(0, M_1) = \lambda(0, M_2)$, then, when M changes from M_1 to M_2,*

(a) *$V_t(x, i)$ becomes higher;*
(b) *$s_t(i)$ gets lower;*
(c) *$V_t'(x, i)$ increases.*

If we increase the maximum number of possible down periods for carrier U, then intuitively, the probability that it will become available after i periods down, $i \leq M_1$, would be smaller. And if this holds, the optimal system cost gets higher and the optimal deliver-down-to levels of the original system become lower.

Proposition 3 *Suppose $\lambda(i)$ is increasing for $i > 0$. Then, for any given $0 < i \leq M$, if $\lambda(i)$ is increased, then the corresponding $s_t(i)$ becomes higher for $t = 1, \ldots, T + 1$.*

Proof We use Implicit Function Theorem to prove the proposition. We take derivative of (15) with respect to $\lambda(i)$ and note that $(\mathsf{E}[V_{t+1}(s_t(i) + D, i + 1)])'_{\lambda(i)} = 0$ because $s_t(i) + D > s_{t+j}(i)$ for $j \geq 0$. As a result,

$$\big(\mathsf{E}[h(s_t(i) + D)]'' + \alpha\lambda(i)\mathsf{E}[V_{t+1}(s_t(i) + D, 0)]'' + \alpha(1 - \lambda(i))$$
$$\mathsf{E}[V_{t+1}(s_t(i) + D, i + 1)]''\big)s_t'(i) = \alpha(\mathsf{E}[V_{t+1}(s_t(i) + D, i + 1)]' - \mathsf{E}[V_{t+1}(s_t(i) + D, 0)]').$$

Because of the convexity of $V_t(\cdot, i)$ and $h(\cdot)$, it is sufficient to show $\mathsf{E}[V_{t+1}(s_t(i) + D, i + 1)]' \geq \mathsf{E}[V_{t+1}(s_t(i) + D, 0)]'$, which has been shown in the proof of Proposition 2. So the proposition follows. □

This proposition implies that, given the state of the carrier U, if the probability that it becomes available in next period gets higher, the manufacturer should have higher optimal deliver-down-to level.

We end this subsection with some numerical examples to illustrate the structural properties we have obtained. The basic parameters for the examples are $M = 5$, $T = 10$, and $\alpha = 0.92$. The examples are generated by alternating the unit shipping cost for either carriers. The random demand in each period is negative binomial distributed with parameters $n = 50$ and $p = 0.3$. The inventory holding and customer waiting cost is $h(x) = 0.1x^2$ for $x \geq 0$. The transition probabilities are $\lambda(0) = 0.5, \lambda(1) = 0.3, \lambda(2) = 0.4, \lambda(3) = 0.5, \lambda(4) = 0.6, \lambda(5) = 1$.

In Table 1, we present the optimal thresholds for the examples. Note that the optimal thresholds are decreasing in the unit shipping cost c_u of unreliable carrier U while are increasing in the unit shipping cost c_r of reliable carrier R. The intuitive explanation is that as the difference of costs between U and R becomes smaller (larger), the manufacturer has less (more) incentive to hold inventory to wait until carrier U becomes available.

3.2 Setup costs

In this subsection, there are fixed transportation costs for both carriers and assume $K_r \geq K_u$. The optimality equation (1) becomes

$$V_t(x_t, i_t) = \min_{x_t \geq y_t \geq 0} \{K(i_t)\mathbf{1}(y_t < x_t) + c(i_t)(x_t - y_t) + \mathsf{E}[h(y_t + D)]$$
$$+ \alpha[\lambda(i_t)\mathsf{E}[V_{t+1}(y_t + D, 0)] + (1 - \lambda(i_t))\mathsf{E}[V_{t+1}(y_t + D, i_t + 1)]]\}, \quad (6)$$

Table 1 Optimal thresholds for uncapacitated carriers without setup costs

$c_u(c_r = 10)$	$s(0)$	$s(1)$	$s(2)$	$s(3)$	$s(4)$	$s(5)$	$c_r(c_u = 2)$	$s(0)$	$s(1)$	$s(2)$	$s(3)$	$s(4)$	$s(5)$
1	0	1	6	10	15	30	6	0	0	0	0	0	6
2	0	0	4	7	11	26	7	0	0	0	0	2	11
3	0	0	2	5	8	21	8	0	0	0	2	5	16
4	0	0	0	3	6	17	9	0	0	1	5	8	21
5	0	0	0	1	3	12	10	0	0	4	7	11	26

in which $K(i_t) = K_u$ if $i_t = 0$ and $K(i_t) = K_r$ otherwise. Assume $V_{T+1}(x_{T+1}, i_{T+1}) = K(i_{T+1})\mathbf{1}(x_{T+1} > 0) + c(i_{T+1})x_{T+1}$.

The proof of the following lemma is given in the Appendix.

Lemma 4 *If there are setup costs for both carriers, for a given i, $V_t(x, i)$ is increasing in x.*

In the remaining analysis, we assume either $K_u \geq \alpha K_r$ or $\mathsf{E}[h'(D)] \geq c_u$. Before characterizing the optimal shipping policy for the manufacturer, we introduce the following definition of $\{K, 0\}$-convexity, which is a simple modification of the K-convexity concept introduced by Scarf (1960) (see also a more general definition in Chen and Simchi-Levi 2003).

Definition 1 A real value function f is called $\{K, 0\}$-convex for $K \geq 0$, if

$$f(\beta x_1 + (1-\beta)x_2) \leq \beta(f(x_1) + K) + (1-\beta)f(x_2)$$

holds for any $x_1 \leq x_2$, $\beta \in [0, 1]$.

Without much abuse of notation, in this section with setup costs, we still let

$$\begin{aligned} J_t(y, i) = -c(i)y + \mathsf{E}[h(y + D)] + \alpha(\lambda(i)\mathsf{E}[V_{t+1}(y + D, 0)] \\ + (1 - \lambda(i))\mathsf{E}[V_{t+1}(y + D, i + 1)]), \end{aligned} \tag{7}$$

then

$$V_t(x_t, i_t) = \min_{x_t \geq y_t \geq 0} \{K(i_t)\mathbf{1}(y_t < x_t) + J_t(y_t, i_t)\} + c(i_t)x_t.$$

We are now ready to present the main result of this section.

Theorem 2 *When there exist setup costs for both carriers, for a given i,*

(a) *$J_t(y, i)$ is a $\{K_r, 0\}$-convex function in $y \geq 0$;*
(b) *$V_t(x, i)$ is a $\{K_r, 0\}$-convex function in $x \geq 0$;*
(c) *There exists a sequence of numbers $0 \leq s_t(i) < S_t(i)$, which is defined as*

$$s_t(i) = \arg\min_{y \geq 0} J_t(y, i) \tag{8}$$

and

$$S_t(i) = \max\{y | J_t(y, i) = J_t(s_t(i), i) + K(i), y \geq s_t(i)\}. \tag{9}$$

For $i = 0, 1, 2, \ldots, M$, the optimal policy is an (s, S)-type of policy, that is,

$$y_t^*(i) = \begin{cases} s_t(i) & \text{if } x_t \geq S_t(i), \\ x_t & \text{otherwise.} \end{cases}$$

With the optimal $(s_t(i), S_t(i))$, the manufacturer ships $x - s_t(i)$ units if the current state of U is i and the inventory level is higher than $S_t(i)$; otherwise, do not ship anything. We note that if $\mathsf{E}[h'(D)] \geq c_u$, then $s_t(0) = 0$.

4 Capacitated carriers

In many applications, the transportation carrier only has a finite shipping capacity for the manufacturer. In this section, carrier U is assumed to have a constant capacity C_u if it is available and carrier R has capacity C_r which can be interpreted as the volume flexibility carrier R offers to the manufacturer. In this case, since U has a limited shipping capacity, in each period, the manufacturer may schedule shipments with both carriers U and R even if U is available. We first assume no setup costs for both carriers. The optimality equations are given by

$$\begin{aligned} V_t(x_t, 0) = \min_{x_t \geq y_t \geq \max\{0, x_t - C_r - C_u\}} &\{c_u(x_t - y_t) + (c_r - c_u)(x_t - y_t - C_u)^+ + \mathsf{E}[h(y_t + D)] \\ &+ \alpha\lambda(0)\mathsf{E}[V_{t+1}(y_t + D, 0)] + \alpha(1 - \lambda(0))\mathsf{E}[V_{t+1}(y_t + D, 1)]\}, \end{aligned} \tag{10}$$

and for $i_t = 1, 2, \ldots, M$,

$$\begin{aligned} V_t(x_t, i_t) = \min_{x_t \geq y_t \geq \max\{x_t - C_r, 0\}} &\{c_r(x_t - y_t) + \mathsf{E}[h(y_t + D)] + \alpha\lambda(i_t)\mathsf{E}[V_{t+1}(y_t + D, 0)] \\ &+ \alpha(1 - \lambda(i_t))\mathsf{E}[V_{t+1}(y_t + D, i_t + 1)]\}. \end{aligned} \tag{11}$$

Recall that y_t denotes the inventory level after shipping.

The proof of the following lemma is parallel to that in the previous section, thus it is omitted.

Lemma 5 *When there are capacity constraints for both carriers, for a given i, $V_t(x, i)$ is increasing and convex in x.*

For $i = 0$, define

$$s_t^1(0) = \arg\min_{y \geq 0} J_t^1(y, 0)$$

where

$$\begin{aligned} J_t^1(y, 0) = &-c_r y + \mathsf{E}[h(y + D)] + \alpha\lambda(0)\mathsf{E}[V_{t+1}(y + D, 0)] \\ &+ \alpha(1 - \lambda(0))\mathsf{E}[V_{t+1}(y + D, 1)], \end{aligned} \tag{12}$$

and

$$s_t^2(0) = \arg\min_{y \geq 0} J_t^2(y, 0)$$

where

$$\begin{aligned} J_t^2(y, 0) = &-c_u y + \mathsf{E}[h(y + D)] + \alpha\lambda(0)\mathsf{E}[V_{t+1}(y + D, 0)] \\ &+ \alpha(1 - \lambda(0))\mathsf{E}[V_{t+1}(y + D, 1)]. \end{aligned} \tag{13}$$

Note that $s_t^1(0) \geq s_t^2(0)$. For $i > 0$, define

$$s_t(i) = \arg\min_{y \geq 0} J_t(y, i),$$

where

$$J_t(y,i) = -c_r y + \mathsf{E}[h(y+D)] + \alpha\lambda(i)\mathsf{E}[V_{t+1}(y+D,0)] \\ + \alpha(1-\lambda(i))\mathsf{E}[V_{t+1}(y+D,i+1)]. \tag{14}$$

The following theorem characterizes the optimal shipping policy for the manufacturer when both carriers have finite shipping capacities.

Theorem 3 *There exist two numbers $s_t^1(0)$ and $s_t^2(0)$and a sequence of number $s_t(i)$ for $i = 1, 2, \ldots, M$. The optimal shipping policy is a modified threshold policy, which is given by, for $i = 0$,*

$$y_t^*(0) = \begin{cases} x_t & \text{if } x_t \le s_t^2(0), \\ s_t^2(0) & \text{if } C_u \ge x_t - s_t^2(0) > 0, \\ x_t - C_u & \text{if } C_u + s_t^1(0) \ge x_t > C_u + s_t^2(0), \\ s_t^1(0) & \text{if } C_u + C_r \ge x_t - s_t^1(0) > C_u, \\ x_t - (C_u + C_r) & \text{otherwise}, \end{cases}$$

for $i = 1, 2, \ldots, M$,

$$y_t^*(i) = \begin{cases} x_t & \text{if } x_t \le s_t(i), \\ s_t(i) & \text{if } C_r \ge x_t - s_t(i) > 0, \\ x_t - C_r & \text{otherwise}. \end{cases}$$

Proof The theorem follows from the convexity of (12, 13 and 14). □

This policy works as follows. At the beginning of each period, if U is available but the initial inventory level is less than $s_t^2(0)$, then ship nothing; if the initial inventory level is higher than $s_t^2(0)$ and $x_t - s_t^2(0)$ is less than C_u, the capacity of U, then ship to reduce the inventory level down to $s_t^2(0)$; if the inventory level is between $s_t^1(0) + C_u$ and $s_t^2(0) + C_u$, use up the shipping capacity of carrier U; if the inventory level is between $C_u + C_r + s_t^1(0)$ and $s_t^1(0) + C_u$, ship to reduce the inventory level down to $s_t^1(0)$; if the inventory level is higher than $s_t^1(0) + C_u + C_r$, use up both carriers' shipping capacities. If carrier U has been down for $i - 1$ periods and the initial inventory level is less than $s_t(i)$, ship nothing; if the inventory level is between $C_r + s_t(i)$ and $s_t(i)$, then ship to reduce the inventory level down to $s_t(i)$; otherwise, use up the capacity of carrier R.

Lemma 6 *(a) For a given C_r, we have, for all t and i, if the carrier U has more capacity, i.e. C_u increases, then*

(i) *$V_t(x,i)$ gets lower;*
(ii) *$s_t^j(0)$ for $j = 1, 2$, and $s_t(i)$ for $i \ge 1$, become higher.*

(b) *For a given C_u, we have, for all t and i, if the carrier R has more capacity, i.e. C_r increases, then*

(i) *$V_t(x,i)$ gets lower;*
(ii) *$s_t^j(0)$ for $j = 1, 2$, and $s_t(i)$ for $i \ge 1$, become higher.*

The lemma basically says that if the shipping capacity of either carrier increases given the other carrier's capacity fixed, the optimal thresholds increase. However, this does not imply that the thresholds will increase when the total capacities of two carriers increase. Intuitively, this is because the thresholds are more sensitive to the unreliable carrier's capacity level than that of the reliable carrier.

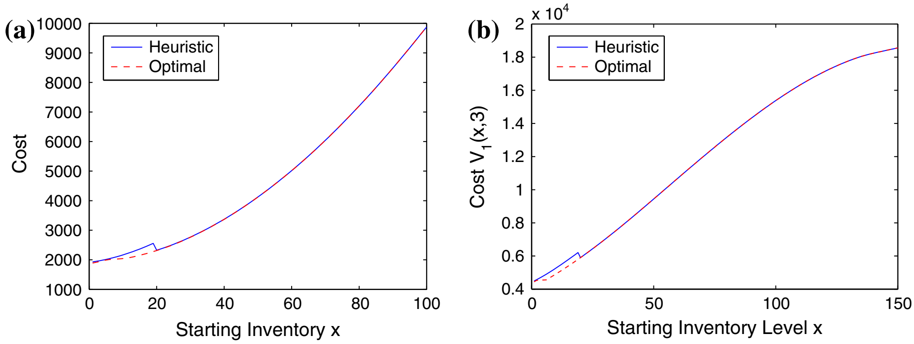

Fig. 1 Heuristic cost versus optimal cost

We conclude this section with some remarks for the case with fixed setup costs as well as capacity constraints for the carriers. The structure of the optimal delivery policy for that case will be very complicated. Even for the model with only one reliable carrier that has setup cost and capacity constraint, the form of the optimal control policy is unknown and can only be partially characterized. Interested readers are referred to Shaoxiang and Lambrecht (1996), Gallego and Scheller-Wolf (2000), and Shaoxiang (2004) on some partial characterizations of the optimal inventory control policy of the classical inventory model with setup costs and capacity constraints.

In light of the optimality of modified base-stock policy and (s, S) policy, we propose a modified $(s_t(i), S_t(i))$ policy as a heuristic and evaluate its performance with some simple numerical examples. The modified $(s_t(i), S_t(i))$ policy works as follows, if the inventory level is higher than $S_t(i)$ then deliver down to $s_t(i)$ or using full delivery capacity; otherwise, deliver nothing. The parameters $S_t(i)$ and $s_t(i)$ are computed without capacity constraints. In Fig. 1, we compare the cost of the heuristic policy with the minimum cost for an example with $K_u = 50$, $K_r = 80$, $c_u = 5$, $c_r = 8$, $M = 5$ and demand is Poisson with $\lambda = 20$. The capacity levels are $C_0 = 30$, $C_1 = 10$ for (a) and $C_0 = 10$, $C_1 = 5$ for (b). As we can see from the figure, the resulting costs of heuristic policy is close to those of the optimal policy especially when the starting inventory level is high.

5 Summary and discussion

In today's global economy, as more companies outsource their transportation/logistics to the third party logistics providers, it is important for the firm to optimally manage its transportation carriers and schedule its shipments. In this paper, we studied a make-to-order production/inventory model with two carriers, one reliable and one unreliable under several different scenarios. We modeled the unreliable carrier as a discrete time Markov process with finite number of states. We characterized the optimal transportation strategies for each scenario. When there is neither setup cost nor capacity constraints, we showed that the total discounted system costs are increasing in the initial inventory level and the optimal threshold is increasing as the number of periods that the unreliable carrier has been down increases. If there exist fixed setup costs for both carriers, we established the optimality of state-dependent (s, S)—type policies. When the transportation carriers are capacitated, we showed that the modified threshold-type policies are optimal and the optimal thresholds are increasing in the capacity level of either carrier.

There are still several related research questions need to be answered. For example, how can the high-cost responsive carrier survive in the current intensified market competition? How do carriers determine their transportation prices and guaranteed delivery leadtime? How does manufacturer charge customers the delivery fees for different delivery leadtimes as different customers have different preferences on prices and leadtimes (See Chen-Ritzo et al. (2005) for an experimental study on how customers tradeoff between cost and leadtimes)? To address the first question, we believe that competition is based on not only cost, but also other factors, such as service level and delivery time, among others, which means that a mathematical model has to be developed that includes all these dimensions of each carriers. We will investigate these in the future research.

We conclude this paper by discussing several extensions.

5.1 General production leadtime

In our paper, we assume the production leadtime is 1. If we relax this assumption and generalize the length of production leadtime to L, for the model with no setup cost and capacity constraint, the problem can be formulated as,

$$\begin{aligned} V_t(x_t, \bar{d}, i_t) = \min_{x_t \geq y_t} \Big\{ & c(i_t)(x_t - y_t) + \mathsf{E}[h(y_t + d_{t-L})] \\ & + \alpha(\lambda(i_t)\mathsf{E}[V_{t+1}(y_t + d_{t-L}, \bar{d}', 0)] \\ & + (1 - \lambda(i_t))\mathsf{E}[V_{t+1}(y_t + d_{t-L}, \bar{d}', i_t + 1)] \Big\}, \end{aligned}$$

in which $\bar{d} = [d_{t-L}, \ldots, d_{t-1}]$ and $\bar{d}' = [d_{t-L+1}, \ldots, D_t]$ are both L-dimension vectors, which represent the realized demand in the past L periods from period t and period $t+1$, respectively. Since only the on-hand stock can be delivered and the system is make-to-order, we need to include the WIP products in pipeline into state of the system. We note that the results in the main context of the paper can all be extended to the general leadtime case, while the optimal policy will become more complicated, depending on the realized demand in the past L periods.

5.2 Make-to-stock system

In this subsection, we consider the make-to-stock system. If we still assume the production leadtime is 1 and let c_p denote the unit production cost, b and h' be the unit backlog (customer waiting) cost and holding cost of inventory respectively, then we can formulate the problem (no setup cost and capacity constraint) as

$$\begin{aligned} V_t(x_t, z_t, i_t) = & \min_{u_t - y_t \geq 0, 0 \leq x_t - y_t \leq z_t} \{c_p(u_t - y_t) + c(i_t)(x_t - y_t) \\ & + \mathsf{E}[b(z_t + D - (x_t - y_t))] + h' y_t \\ & + \alpha\lambda(i_t)\mathsf{E}[V_{t+1}(u_t, z_t + D - (x_t - y_t), 0)] \\ & + \alpha(1 - \lambda(i_t))\mathsf{E}[V_{t+1}(u_t, z_t + D - (x_t - y_t), i_{t+1})]\}, \end{aligned}$$

in which x_t is the starting inventory level while z_t is the total purchase order waiting to be delivered. The first term in the braces is the production cost, the second term is the shipping cost, the third term is the customer waiting cost and the fourth term is the inventory holding cost. It is clear that $V_t(x_t, z_t, i_t)$ is jointly convex in x_t and z_t and the optimal production policy is base-stock type policies, which depend on the state i_t of the unreliable carrier, starting inventory level x_t and the number of waiting customers z_t. The shipping policy is still deliver-down-to type policies that also depend on the state of unreliable carrier and starting inventory level.

5.3 Infinite planning horizon

The model can be extended to the case with an infinite planning horizon. By following the standard arguments (see Ross 1983), we can show that, under mild technical conditions, the results we have derived in previous sections can all be extended to the infinite horizon case.

Acknowledgements We thank two anonymous referees for their helpful comments that improve the presentation of the paper. This work was done when the first two authors were visiting the Department of Industrial Engineering and Logistics Management of Hong Kong University of Science and Technology and is supported in part by Hong Kong RGC grant HKUST 6145/03E. The first author is partially supported by CUHK Direct Grant under No. 2050372. The second author is also partially supported by NSF under DMI-0200306, and a grant from National Natural Science Foundation of China under No.70228001.

Appendix

In this appendix, we provide the proofs of Propositions 1 and 2, Lemmas 4 and 6, and Theorem 2. Unless otherwise noted, in the following proof, we use $\mathsf{E}[f'(y)]$ instead of $\mathsf{E}[f(y)]'$ as the expectation and derivative are assumed interchangeable for a continuous function $f(y)$, which follows from Leibniz's rule.

Proof of Proposition 1. We prove this proposition by induction on t. It is clearly true for $t = T + 1$. Suppose it is true for $t = k + 1$. For any period $t = k$, if $i \geq 1$, $s_k(i)$ is the minimizer of (3) such that

$$\begin{aligned} &-c_r + \mathsf{E}[h'(s_k(i) + D)] + \alpha\lambda(i)\mathsf{E}[V'_{k+1}(s_k(i) + D, 0)] \\ &\quad + \alpha(1 - \lambda(i))\mathsf{E}[V'_{k+1}(s_k(i) + D, i + 1)] = 0. \end{aligned} \tag{15}$$

It suffices to show that $J'_k(s_k(i), i - 1) \geq 0$. Note that, for $i > 1$,

$$\begin{aligned} &J'_k(s_k(i), i - 1) \\ &\quad = -c_r + \mathsf{E}[h(s_k(i) + D)]' + \alpha\lambda(i - 1)\mathsf{E}[V_{k+1}(s_k(i) + D, 0)]' \\ &\qquad + \alpha(1 - \lambda(i - 1))\mathsf{E}[V_{k+1}(s_k(i) + D, i)]' \\ &\quad = -(\alpha\lambda(i)\mathsf{E}[V_{k+1}(s_k(i) + D, 0)]' + \alpha(1 - \lambda(i))\mathsf{E}[V'_{k+1}(s_k(i) + D, i + 1)]) \\ &\qquad + \alpha\lambda(i - 1)\mathsf{E}[V_{k+1}(s_k(i) + D, 0)]' + \alpha(1 - \lambda(i - 1))\mathsf{E}[V'_{k+1}(s_k(i) + D, i)] \\ &\quad = \alpha(\lambda(i - 1) - \lambda(i))c_u + \alpha(1 - \lambda(i - 1))\mathsf{E}[V_{k+1}(s_k(i) + D, i)]' \\ &\qquad - \alpha(1 - \lambda(i))\mathsf{E}[V'_{k+1}(s_k(i) + D, i + 1)] \\ &\quad = \alpha(\lambda(i - 1) - \lambda(i))c_u + \alpha(1 - \lambda(i - 1))c_r - \alpha(1 - \lambda(i))\mathsf{E}[V'_{k+1}(s_k(i) + D, i + 1)] \\ &\quad \geq \alpha(\lambda(i - 1) - \lambda(i))c_u + \alpha(1 - \lambda(i - 1))c_r - \alpha(1 - \lambda(i))c_r \\ &\quad = \alpha(\lambda(i) - \lambda(i - 1))(c_r - c_u) \\ &\quad \geq 0, \end{aligned}$$

in which the second equality follows from (15), the third equality follows from $V'_{k+1}(s_k(i) + D, 0) = c_u$ for any sample path D because $s_k(i) + D \geq s_{k+1}(i) + D \geq s_{k+1}(0)$ by the inductive assumption and Lemma 3, and the fourth equality follows from $V'_{k+1}(s_k(i) + D, i) = c_r$ because $s_k(i) + D \geq s_{k+1}(i)$ for any sample path D; the first inequality follows from

$V_k'(x, i) \le c_r$ because of the convexity of $V_k(x, i)$ and the second inequality is due to $\lambda(i) \ge \lambda(i-1)$. Therefore, $s_t(i) \ge s_t(i-1)$.

If $i = 1$, we have

$$\begin{aligned}
&J_k'(s_k(1), 0)\\
&\quad = -c_u + \mathsf{E}[h(s_k(1)+D)]' + \alpha\lambda(0)\mathsf{E}[V_{k+1}'(s_k(1)+D, 0)] + \alpha(1-\lambda(0))\mathsf{E}[V_{k+1}'(s_k(1)+D, 1)]\\
&\quad = c_r - c_u + \alpha(\lambda(0) - \lambda(1))c_u + \alpha(1-\lambda(0))\mathsf{E}[V_{k+1}'(s_k(1) + D, 1)]\\
&\qquad -\alpha(1-\lambda(1))\mathsf{E}[V_{k+1}'(s_k(1) + D, 2)]\\
&\quad \ge c_r - c_u + \alpha(\lambda(0) - \lambda(1))c_u + \alpha(1-\lambda(0))c_r - \alpha(1-\lambda(1))c_r\\
&\quad = c_r - c_u + \alpha(\lambda(1) - \lambda(0))(c_r - c_u)\\
&\quad \ge 0,
\end{aligned}$$

where the second equality follows from (15) for $i = 1$, the first inequality again follows from that $V_k'(x, i) \le c_r$ for $i > 0$, and the last inequality holds because $|\alpha(\lambda(1) - \lambda(0))| < 1$ which does not depend on the relationship between $\lambda(1)$ and $\lambda(0)$. Thus we complete the proof.

Proof of Proposition 2. We use notation $V_t(x, i, M)$, $J_t(y, i, M)$, $\lambda(i, M)$ and $s_t(i, M)$ to emphasize the dependency on M. We prove this result by induction on t. From the initial condition, it is clear that the proposition is valid for $t = T + 1$. Suppose (a), (b) and (c) are all true for $t = k + 1$. For period $t = k$, we first prove part (b). Note that $M_1 \le M_2$ and for $j = 1, 2$, $s_k(i, M_j)$ is the minimizer of

$$\begin{aligned}
J_k(y, i, M_j) = -c(i)y + \mathsf{E}[h(y + D)] + \alpha\lambda(i, M_j)\mathsf{E}[V_{k+1}(y + D, 0, M_j)]\\
+\alpha(1-\lambda(i, M_j))\mathsf{E}[V_{k+1}(y + D, i + 1, M_j)].
\end{aligned} \tag{16}$$

It is sufficient to show that

$$J_k'(s_k(i, M_2), i, M_1) \le 0.$$

By the inductive assumption of part (c), $V_{k+1}'(x, i, M_2) \ge V_{k+1}'(x, i, M_1)$, so

$$\begin{aligned}
&J_k'(s_k(i, M_2), i, M_1)\\
&\quad \le \alpha(\lambda(i, M_1) - \lambda(i, M_2))\Big(\mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, 0, M_2)]\\
&\qquad -\mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, i + 1, M_2)]\Big).
\end{aligned}$$

Because $\lambda(i, M_2) \le \lambda(i, M_1)$, the above function is negative if $\mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, 0, M_2)] \le \mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, i + 1, M_2)]$. From (5) and Proposition 1, we know $-c_r + \mathsf{E}[h'(s_k(i, M_2) + D)] + \alpha c_u \le 0$. Furthermore, for $i \ge 1$,

$$\begin{aligned}
&J_k'(s_k(i, M_2), i, M_2)\\
&\quad = -c(i) + \mathsf{E}[h'(s_k(i, M_2) + D)] + \alpha\mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, 0, M_2)]\\
&\qquad +\alpha(1-\lambda(i, M_2))[\mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, i + 1, M_2)]\\
&\qquad -\mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, 0, M_2)]]\\
&\quad = -c_r + \mathsf{E}[h'(s_k(i, M_2) + D)] + \alpha c_u\\
&\qquad +\alpha(1-\lambda(i, M_2))[\mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, i + 1, M_2)]\\
&\qquad -\mathsf{E}[V_{k+1}'(s_k(i, M_2) + D, 0, M_2)]]\\
&\quad = 0.
\end{aligned}$$

Therefore, $\mathsf{E}[V'_{k+1}(s_k(i, M_2) + D, i + 1, M_2)] - \mathsf{E}[V'_{k+1}(s_k(i, M_2) + D, 0, M_2)] \geq 0$ for $i \geq 1$. For $i = 0$, note that, by inductive assumption of part (a) and $\lambda(0, M_2) = \lambda(0, M_1)$,

$$\begin{aligned}
&J'_k(s_k(0, M_2), 0, M_1)\\
&\quad = -c_u + \mathsf{E}[h'(s_k(0, M_2) + D)] + \alpha\lambda(0, M_1)c_u + \alpha(1 - \lambda(0, M_1))\\
&\qquad \times \mathsf{E}[V'_{k+1}(s_k(0, M_2) + D, 1, M_1)]\\
&\quad \leq -c_u + \mathsf{E}[h'(s_k(0, M_2) + D)] + \alpha\lambda(0, M_2)c_u + \alpha(1 - \lambda(0, M_2))\\
&\qquad \times \mathsf{E}[V'_{k+1}(s_k(0, M_2) + D, 1, M_2)]\\
&\quad = 0
\end{aligned}$$

Hence, $s_k(i, M_2) \leq s_k(i, M_1)$ for all $i \leq M_1$ and part (b) is proved.

For (a), we need to show, for $x^1 \geq x^2$ and

$$V_k(x^1, i, M_2) + V_k(x^2, i, M_1) \geq V_k(x^2, i, M_2) + V_k(x^1, i, M_1).$$

To prove above inequality, we need to discuss several cases. In the following, we use one case to illustrate the proof of above inequality. All other cases can be similarly proved. From (c), we know $s_k(i, M_2) \leq s_k(i, M_1)$. If $x^1 \geq s_k(i, M_1) \geq x^2 \geq s_k(i, M_2)$,

$$\begin{aligned}
&V_k(x^1, i, M_2) - V_k(x^2, i, M_2)\\
&\quad = c(i)(x^1 - x^2)\\
&\quad \geq c(i)(x^1 - x^2) + (J_k(s_k(i, M_1), i, M_1) - J_k(x^2, i, M_1))\\
&\quad = V_k(x^1, i, M_1) - V_k(x^2, i, M_1),
\end{aligned}$$

where the inequality follows from the optimality of $s_k(i, M_1)$.

For part (c), since it can be easily shown by the inductive assumption, so we skip the proof here.

Proof of Lemma 4. For ease of exposition, we rewrite the optimality equation equivalently as follows,

$$V_t(x, i) = \min_{y+Q=x, y\geq 0, Q\geq 0} \{K(i)\mathbf{1}(y < x) + f_1(Q, i) + f_2(y, i)\},$$

where $f_1(Q, i) = c(i)Q$ and $f_2(y, i) = \mathsf{E}[h(y + D)] + \alpha[\lambda(i)\mathsf{E}[V_{t+1}(y + D, 0)] + (1 - \lambda(i))\mathsf{E}[V_{t+1}(y + D, i + 1)]$.

We prove this lemma by induction on t. It is obviously true for $t = T + 1$. Suppose the lemma is true for $t = k + 1$. We next show it is true for $t = k$. Note that both $f_1(\cdot, i)$ and $f_2(\cdot, i)$ are increasing functions. Let $x_2 > x_1$ and Q_1, y_1 and Q_2, y_2 denote the optimal Q and y when the starting inventory level is x_1 and x_2, respectively.

Case 1, $Q_1 = 0$, $y_1 = x_1$,

$$V_k(x_1, i) = f_1(0, i) + f_2(x_1, i).$$

Subcase 1, $Q_2 = 0$, $y_2 = x_2$,

$$V_k(x_2, i) = f_1(0, i) + f_2(x_2, i) \geq f_1(0, i) + f_2(x_1, i).$$

The inequality follows from that $f_2(\cdot, i)$ is nondecreasing.

Subcase 2, $0 < Q_2 \leq x_1$, $y_2 = x_2 - Q_2$,

$$\begin{aligned}
V_k(x_2, i) &= K(i) + f_1(Q_2, i) + f_2(x_2 - Q_2, i)\\
&\geq f_1(0, i) + f_2(x_1, i) + f_2(x_2 - Q_2, i) - f_2(x_1 - Q_2, i)\\
&\geq f_1(0, i) + f_2(x_1, i).
\end{aligned}$$

The first inequality follows from the optimality of $(0, x_1)$ and the second inequality again from nondecreasingness of $f_2(\cdot, i)$.
Subcase 3, $x_1 < Q_2 \le x_2$, $y_2 = x_2 - Q_2$,

$$\begin{aligned} V_k(x_2, i) &= K(i) + f_1(Q_2, i) + f_2(x_2 - Q_2, i) \\ &\ge K(i) + f_1(x_1, i) + f_2(0, i) + f_2(x_2 - Q_2, i) - f_2(0, i) \\ &\ge f_1(0, i) + f_2(x_1, i) + f_2(x_2 - Q_2, i) - f_2(0, i) \\ &\ge f_1(0, i) + f_2(x_1, i), \end{aligned}$$

where the first inequality follows from that $f_1(Q, i)$ is nondecreasing.
Case 2, $0 < Q_1 \le x_1$, $y_1 = x_1 - Q_1$,

$$V_k(x_1, i) = K(i) + f_1(Q_1) + f_2(x_1 - Q_1, i).$$

In this case, the optimal strategy is to ship Q_1 units when the starting inventory level is x_1.
Subcase 1, $Q_2 = 0$, $y_2 = x_2$,

$$\begin{aligned} V_k(x_2, i) &= f_1(0, i) + f_2(x_2, i) \\ &\ge K(i) + f_1(Q_1, i) + f_2(x_1 - Q_1, i) + f_2(x_2, i) - f_2(x_1, i) \\ &\ge K(i) + f_1(Q_1, i) + f_2(x_1 - Q_1, i). \end{aligned}$$

The first inequality holds because $Q = 0$, $y = x_1$ is a feasible policy and it cannot outperform the optimal policy $Q = Q_1$, $y = y_1$.
Subcase 2, $0 < Q_2 \le x_1$, $y_2 = x_2 - Q_2$,

$$\begin{aligned} V_k(x_2, i) &= K(i) + f_1(Q_2, i) + f_2(x_2 - Q_2, i) \\ &\ge K(i) + f_1(Q_1, i) + f_2(x_1 - Q_1, i) + f_2(x_2 - Q_2, i) - f_2(x_1 - Q_2, i) \\ &\ge K(i) + f_1(Q_1, i) + f_2(x_1 - Q_1, i). \end{aligned}$$

Again the first inequality follows from that the feasibility of policy $Q = Q_2$, $y = x_1 - Q_2$ and the optimality of $Q = Q_1$, $y = y_1$.
Subcase 3, $x_1 < Q_2 \le x_2$, $y_2 = x_2 - Q_2$,

$$\begin{aligned} V_k(x_2, i) &= K(i) + f_1(Q_2, i) + f_2(x_2 - Q_2, i) \\ &\ge K(i) + f_1(x_1, i) + f_2(0, i) + f_2(x_2 - Q_2, i) - f_2(0, i) \\ &\ge K(i) + f_1(Q_1, i) + f_2(x_1 - Q_1, i) + f_2(x_2 - Q_2, i) - f_2(0, i) \\ &\ge K(i) + f_1(Q_1, i) + f_2(x_1 - Q_1, i). \end{aligned}$$

So we have proved that $V_t(x, i)$ is nondecreasing in x.
Proof of Theorem 2. We prove Theorem 2 by induction on t. Recall that $V_{T+1}(x_{T+1}, i_{T+1}) = K(i_{T+1})\mathbf{1}(x_{T+1} > 0) + c(i_{T+1})x_{T+1}$, so the lemma is true for $t = T + 1$. Suppose it is true for $t = k + 1$ and let $0 \le y_1 \le y_2$, then for $t = k$ and any $i = 0, 1, \ldots, M$, we have

$$\begin{aligned} \mathsf{E}[V_{k+1}(\beta y_1 + (1-\beta) y_2 + D, i)] \le \beta(&\mathsf{E}[V_{k+1}(y_1 + D, i)] + K_r) \\ &+ (1-\beta)\mathsf{E}[V_{k+1}(y_2 + D, i)]. \end{aligned}$$

Because $h(\cdot)$ is a convex function,

$$
\begin{aligned}
J_k(\beta y_1 + (1-\beta)y_2, i) \le \beta(-c(i)y_1 + \mathsf{E}[h(y_1+D)]) + (1-\beta)(-c(i)y_2 + \mathsf{E}[h(y_2+D)])\\
&+ \alpha\lambda(i)[\beta(\mathsf{E}[V_{k+1}(y_1+D,0)] + K_r)\\
&+ (1-\beta)\mathsf{E}[V_{k+1}(y_2+D,0)]]\\
&+ \alpha(1-\lambda(i))[\beta(\mathsf{E}[V_{k+1}(y_1+D,i+1)] + K_r)\\
&+ (1-\beta)\mathsf{E}[V_{k+1}(y_2+D,i+1)]]\\
&= \beta J_k(y_1,i) + (1-\beta)J_k(y_2,i) + \alpha\beta K_r
\end{aligned}
$$

Hence $J_k(y,i)$ is $\{\alpha K_r, 0\}$-convex and so it is $\{K_r, 0\}$-convex.

For part (c), we first show the case $i = 0$. If $K_u \ge \alpha K_r$, because $J_k(y,i)$ is $\{\alpha K_r, 0\}$-convex, then for $y \ge S_k(0)$, $J_k(y,i)$ is increasing. Hence, when $i = 0$, it is optimal for the manufacturer to deliver down to $s_k(0)$ if $x \ge S_k(0)$. If $\mathsf{E}[h'(D)] \ge c_u$, because $V(\cdot, i)$ is increasing and $h(\cdot)$ is convex, then $s_k(0) = 0$ and it is optimal to deliver down to $s_k(0)$ if $x \ge S_k(0)$. So part (c) holds for $i = 0$. For $i > 0$, from the $\{\alpha K_r, 0\}$-convexity, it is clear that it is optimal to deliver down to $s_k(i)$ when $x \ge S_k(i)$, otherwise do not ship anything. Thus, (c) is valid for all i.

In what follows we ignore the linear term $c(i)x$ for simplicity and show $V_k(x,i)$ is $\{K_r, 0\}$-convex by discussing several different cases. Let $x_1 \le x_2$.

Case 1, $S_k(i) \le x_1 \le x_2$, then $\beta x_1 + (1-\beta)x_2 \ge S_k(i)$ and

$$
\begin{aligned}
V_k(\beta x_1 + (1-\beta)x_2, i) &= J_k(s_k(i), i) + K(i)\\
&= \beta(J_k(s_k(i), i) + K(i)) + (1-\beta)(J_k(s_k(i), i) + K(i))\\
&= \beta V_k(x_1, i) + (1-\beta)V_k(x_2, i)\\
&\le \beta(V_k(x_1, i) + K_r) + (1-\beta)V_k(x_2, i).
\end{aligned}
$$

Case 2, $s_k(i) \le x_1 < S_k(i) \le x_2$.

Subcase 1, $\beta x_1 + (1-\beta)x_2 > S_k(i)$,

$$
\begin{aligned}
V_k(\beta x_1 + (1-\beta)x_2, i) &= J_k(s_k(i), i) + K(i)\\
&= \beta(J_k(s_k(i), i) + K(i)) + (1-\beta)(J_k(s_k(i), i) + K(i))\\
&\le \beta(V_k(x_1, i) + K_r) + (1-\beta)V_k(x_2, i),
\end{aligned}
$$

in which the inequality follows from that fact that $s_k(i)$ is the minimizer of $J_k(y,i)$ and the optimality of $(s_k(i), S_k(i))$ policy.

Subcase 2, $s_k(i) \le \beta x_1 + (1-\beta)x_2 \le S_k(i)$,

$$
\begin{aligned}
V_k(\beta x_1 + (1-\beta)x_2, i) &\le J_k(s_k(i), i) + K_r\\
&= \beta(J_k(s_k(i), i) + K_r) + (1-\beta)(J_k(s_k(i), i) + K_r)\\
&\le \beta(V_k(x_1, i) + K_r) + (1-\beta)V_k(x_2, i).
\end{aligned}
$$

Case 3, $x_1 \le s_k(i) < S_k(i) \le x_2$.

Subcase 1, $\beta x_1 + (1-\beta)x_2 > s_k(i)$, the proof is similar to the previous case, so we skip here.

Subcase 2, $\beta x_1 + (1-\beta)x_2 < s_k(i)$, then there exists $\xi \le \beta$, s.t. $\beta x_1 + (1-\beta)x_2 = \xi x_1 + (1-\xi)s_k(i)$,

$$\begin{aligned}
V_k(\beta x_1 + (1-\beta)x_2, i) &\le J_k(\beta x_1 + (1-\beta)x_2, i) \\
&= J_k(\xi x_1 + (1-\xi)s_k(i), i) \\
&\le \xi(J_k(x_1, i) + K_r) + (1-\xi)J_k(s_k(i), i) \\
&= \beta(V_k(x_1, i) + K_r) + (1-\beta)J_k(s_k(i), i) \\
&\quad + (\beta - \xi)(J_k(s_k(i), i) - J_k(x_1, i) - K_r) \\
&\le \beta(V_k(x_1, i) + K_r) + (1-\beta)V_k(x_2, i),
\end{aligned}$$

where the second equality follows from that $V_k(x_1, i) = J_k(x_1, i)$ and the last inequality follows from $J_k(s_k(i), i) - J_k(x_1, i) < 0$ and $V_k(x_2, i) \ge J_k(s_k(i), i)$.
Case 4, $s_k(i) \le x_1 \le x_2 < S_k(i)$,

$$\begin{aligned}
V_k(\beta x_1 + (1-\beta)x_2, i) &\le J_k(\beta x_1 + (1-\beta)x_2, i) \\
&\le \beta(J_k(x_1, i) + K_r) + (1-\beta)J_k(x_2, i) \\
&= \beta(V_k(x_1, i) + K_r) + (1-\beta)V_k(x_2, i).
\end{aligned}$$

Case 5, $x_1 < s_k(i) < x_2 < S_k(i)$, this case can be similarly proved as Case 4.
Case 6, $x_1 < x_2 < s_k(i) < S_k(i)$, this case can be similarly proved as Case 4.

Thus, we complete the proof.

Proof of Lemma 6. We only prove part (a) as the proof for part (b) is parallel. In the following discussion, to emphasize the dependency on the capacity, we use $V_t(x, i, C)$, $J_t(y, i, C)$, $s_t^j(0, C)$ for $j = 1, 2$ and $s_t(i, C)$ to denote the optimal value function and optimal thresholds for given capacity C and the derivatives are all with respect to the first variable of the function.

(1) Because the optimal policy for the problem with capacity C_u^1 is feasible for the problem with capacity C_u^2, (i) follows.
(2) We prove it by induction on t. For $t = k+1$, suppose $V'_{k+1}(x, i, C_u^2) \le V'_{k+1}(x, i, C_u^1)$, then $J_k^{j'}(y, 0, C_u^2) \le J_k^{j'}(y, 0, C_u^1)$ for $j = 1, 2$, and $J'_k(y, i, C_u^2) \le J'_k(y, i, C_u^1)$ for $i > 1$, which imply $s_k^j(0, C_u^2) \ge s_k^j(0, C_u^1)$ for $j = 1, 2$ and $s_k(i, C_u^2) \ge s_k(i, C_u^1)$.

For $t = k$, we provide the detailed proof for the case that $i = 0$ and $s_k^2(0, C_u^1) + C_u^1 \le s_k^2(0, C_u^2)$ while the proof for other cases are similar and we leave it for reader.
Case 1, $x < s_k^2(0, C_u^1) < s_k^2(0, C_u^1) + C_u^1 \le s_k^2(0, C_u^2)$,

$$V'_k(x, 0, C_u^2) = c_u + J_k^{2'}(x, 0, C_u^2) \le c_u + J_k^{2'}(x, 0, C_u^1) = V'_k(x, 0, C_u^2).$$

Case 2, $s_k^2(0, C_u^1) \le x < s_k^2(0, C_u^1) + C_u^1 \le s_k^2(0, C_u^2)$,

$$V'_k(x, 0, C_u^2) = c_u + J_k^{2'}(x, 0, C_u^2) \le c_u = V'_k(x, 0, C_u^1).$$

Case 3, $s_k^2(0, C_u^1) < s_k^2(0, C_u^1) + C_u^1 \le x < s_k^2(0, C_u^2)$.
Subcase 1, $x < s_k^1(0, C_u^1) + C_u^1$,

$$V'_k(x, 0, C_u^2) = c_u + J_k^{1'}(x, 0, C_u^2) \le c_u \le c_u + J_k^{2'}(x - C_u^1, 0, C_u^1) = V'_k(x, 0, C_u^1),$$

where the last inequality follows from $J_k^{1'}(x - C_u^1, 0, C_u^1) \ge 0$.
Subcase 2, $s_k^1(0, C_u^1) + C_1^1 \le x < s_k^1(0, C_u^1) + C_1^1 + C_r$,

$$V'_k(x, 0, C_u^2) = c_u + J_k^{1'}(x, 0, C_u^2) \le c_u \le c_r + J_k^{1'}(s_k^1(0, C_u^1), 0, C_u^1) = V'_k(x, 0, C_u^1).$$

Subcase 3, $s_k^1(0, C_u^1) + C_1^1 + C_r \le x$,

$$V_k'(x, 0, C_u^2) = c_u + J_k^{1'}(x, 0, C_u^2) \le c_u \le c_r + J_k^{1'}(x - (C_1^1 + C_r), 0, C_u^1) = V_k'(x, 0, C_u^1).$$

Case 4, $s_k^2(0, C_u^2) \le x < s_k^2(0, C_u^2) + C_u^2$.
Subcase 1, $x < s_k^1(0, C_u^1) + C_u^1$,

$$V_k'(x, 0, C_u^2) = c_u \le c_u + J_k^{2'}(x - C_u^1, 0, C_u^1) = V_k'(x, 0, C_u^1).$$

Subcase 2, $s_k^1(0) + C_u \le x < s_k^1(0, C_u^1) + C_1^1 + C_r$,

$$V_k'(x, 0, C_u^2) = c_u \le c_r + J_k^{1'}(s_k^1(0, C_u^1), 0, C_u^1) = V_k'(x, 0, C_u^1).$$

Subcase 3, $s_k^1(0, C_u^1) + C_1^1 + C_r \le x$,

$$V_k'(x, 0, C_u^2) = c_u \le c_r + J_k^{1'}(x - C_u^1 - C_r, 0, C_u^1) = V_k'(x, 0, C_u^1).$$

Case 5, $s_k^2(0, C_u^2) + C_u^2 \le x < s_k^1(0, C_u^2) + C_u^2$.
Subcase 1, $x < s_k^1(0, C_u^1) + C_u^1$,

$$\begin{aligned}V_k'(x, 0, C_u^2) &= c_u + J_k^{2'}(x - C_u^2, 0, C_u^2) \le c_u + J_k^{2'}(x - C_u^1, 0, C_u^2)\\ &\le c_u + J_k^{2'}(x - C_u^1, 0, C_u^1) = V_k'(x, 0, C_u^1),\end{aligned}$$

where the second inequality follows from the inductive assumption.
Subcase 2, $s_k^1(0, C_u^1) + C_u^1 \le x < s_k^1(0, C_u^1) + C_u^1 + C_r$,

$$V_k'(x, 0, C_u^2) = c_r + J_k^{1'}(x - C_u^2, 0, C_u^2) \le c_r = c_r + J_k^{1'}(s_k^1(0, C_u^1), 0, C_u^1) = V_k'(x, 0, C_u^1).$$

Subcase 3, $s_k^1(0, C_u^1) + C_u^1 + C_r \le x$,

$$\begin{aligned}V_k'(x, 0, C_u^2) &= c_r + J_k^{1'}(x - C_u^2, 0, C_u^2) \le c_r \le c_r\\ &\quad + J_k^{1'}(x - C_u^1 - C_r, 0, C_u^1) = V_k'(x, 0, C_u^1).\end{aligned}$$

Case 6, $s_k^1(0, C_u^2) + C_u^2 \le x < s_k^1(0, C_u^2) + C_u^2 + C_r$.
Subcase 1, $s_k^1(0, C_u^1) + C_u^1 \le x < s_k^1(0, C_u^1) + C_u^1 + C_r$,

$$V_k'(x, 0, C_u^2) = c_r + J_k^{1'}(s_k^1(0, C_u^2), 0, C_u^2) = c_r = c_r + J_k^{1'}(s_k^1(0, C_u^1), 0, C_u^1) = V_k'(x, 0, C_u^1).$$

Subcase 2, $s_k^1(0, C_u^1) + C_u^1 + C_r \le x$,

$$\begin{aligned}V_k'(x, 0, C_u^2) &= c_r + J_k^{1'}(s_k^1(0, C_u^2), 0, C_u^2) = c_r \le c_r\\ &\quad + J_k^{1'}(x - C_u^1 - C_r, 0, C_u^1) = V_k'(x, 0, C_u^1).\end{aligned}$$

Case 7, $s_k^1(0, C_u^2) + C_u^2 + C_r \le x$, then $s_k^1(0, C_u^1) + C_u^1 + C_r \le x$,

$$\begin{aligned}V_k'(x, 0, C_u^2) &= c_r + J_k^{1'}(x - C_u^2 - C_r, 0, C_u^2)\\ &\le c_r + J_k^{1'}(x - C_u^1 - C_r, 0, C_u^2)\\ &\le c_r + J_k^{1'}(x - C_u^1 - C_r, 0, C_u^1)\\ &= V_k'(x, 0, C_u^1).\end{aligned}$$

Thus, we have proved (ii) of part (a) if $s_k^2(0, C_u^1) + C_u^1 \le s_k^2(0, C_u^2)$. We can similarly prove the case if $s_k^2(0, C_u^1) + C_u^1 > s_k^2(0, C_u^2)$.

For $i > 0$, we can similarly prove the result by discussing two cases, i.e., $s_k(i, C_r^1) + C_r^1 \le s_k(i, C_r^2)$ and $s_k(i, C_r^1) + C_r^1 > s_k(i, C_r^2)$.

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